FORMULÁRIO MÉTODOS NUMÉRICOS

Método da secante:

$$x_{m+1} = x_m - f(x_m)\frac{x_m - x_{m-1}}{f(x_m) - f(x_{m-1})}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(\eta_m)}e_{m-1}e_m$$
$$\eta_m \in \text{int}(x_{m-1}, x_m) \quad \xi_m \in \text{int}(x_{m-1}, x_m, z)$$

Método de Newton:

$$x_{m+1} = x_m - \frac{f(x_m)}{f'(x_m)}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(x_m)}e_m^2 \qquad \xi_m \in \text{int}(z, x_m)$$

Método do ponto fixo:

$$x_{m+1} = g(x_m)$$

$$|e_{m+1}| \leq L|e_m|$$

$$|e_{m+1}| \leq \frac{L}{1-L}|x_{m+1} - x_m|$$

$$e_{m+1} = \frac{(-1)^{p-1}g^{(p)}(\xi_m)}{p!}e_m^p \qquad \xi_m \in \text{int}(z, x_m)$$
$$g^{(r)}(z) = 0 \qquad r = 0, \ldots, p-1 \qquad g^{(p)}(z) \neq 0$$

Método de Doolittle:

$$l_{ik} = (a_{ik} - \sum_{r=1}^{k-1} l_{ir}u_{rk})/u_{kk}$$

$$u_{kj} = a_{kj} - \sum_{r=1}^{k-1} l_{kr}u_{rj}$$

Método de Cholesky:

$$l_{kk} = \sqrt{a_{kk} - \sum_{r=1}^{k-1} l_{kr}^2}$$

$$l_{ik} = (a_{ik} - \sum_{r=1}^{k-1} l_{ir}l_{kr})/l_{kk}$$

Método Eliminação Gauss

$$m_{ik} = \frac{a_{ik}}{a_{kk}}, \quad E_i - m_{ik}E_k \longrightarrow E_i$$

Método Householder:

$$\begin{cases} \alpha = -\,sgn(a_{k+1,k}^{(k)})\left(\sum_{j=k+1}^{n}(a_{jk}^{(k)})^2\right)^{1/2} \\ r = \left(\frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\, a_{k+1,k}^{(k)}\right)^{1/2}, \\ w_1^{(k)} = w_2^{(k)} = \cdots = w_k^{(k)} = 0\,, \\ w_{k+1}^{(k)} = \frac{a_{k+1,k}^{(k)} - \alpha}{2r}\,, \\ w_j^{(k)} = \frac{a_{jk}^{(k)}}{2r}\,, \text{ para cada } j = k+2, k+3, \ldots, n \\ P^{(k)} = I - 2\mathbf{w}^{(k)} \cdot (\mathbf{w}^{(k)})^t \\ A^{(k+1)} = P^{(k)}A^{(k)}P^{(k)} \end{cases}$$

Métodos Iterativos Sist. Lineares

$$\mathbf{M}\mathbf{x}^{(k+1)} = \mathbf{b} - \mathbf{N}\mathbf{x}^{(k)}$$

$$\mathbf{x} - \mathbf{x}^{(k+1)} = \mathbf{C}(\mathbf{x} - \mathbf{x}^{(k)})$$
$$\mathbf{C} = -\mathbf{M}^{-1}\mathbf{N}$$

Método de Jacobi: $C_{\text{J}} = -D^{-1}(L+U)$

$$x_i^{(k+1)} = \left(b_i - \sum_{j=1, j\neq i}^{n} a_{ij}x_j^{(k)}\right)/a_{ii}$$

Método de Gauss-Seidel: $C_{\text{GS}} = -(L+D)^{-1}U$

$$x_i^{(k+1)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right)/a_{ii}$$

$$\|\mathbf{x} - \mathbf{x}^{(k+1)}\| \leq \frac{\|C\|}{1 - \|C\|}\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|$$

$$\|\mathbf{x} - \mathbf{x}^{(k+1)}\| \leq \|C\|\ \|\mathbf{x} - \mathbf{x}^{(k)}\|$$

Métodos Iterativos para Sist. Não-Lineares
Método de Newton:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)}, \qquad \mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)})$$